Solvay Brussels School
Economics & Management

ULB

Risk Management and Governance
Credit derivatives

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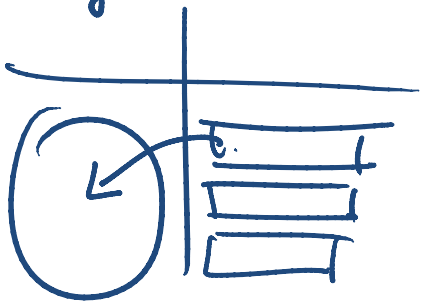
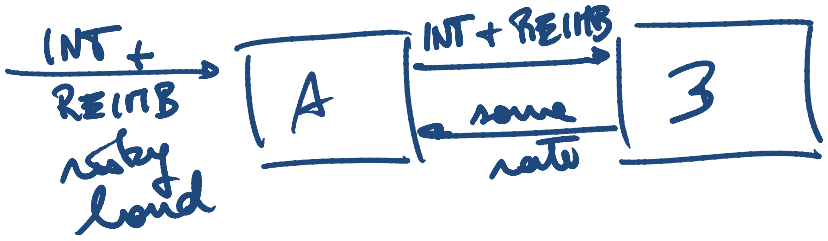
Credit Derivatives ≈ Credit hedging ≈ insurance

- Derivatives where the payoff depends on the credit quality of a company or sovereign entity
- The market started to grow fast in the late 1990s
- By 2005 notional principal totaled $12 trillion
- Main
  - CDS → Credit Default Swap (premium against a potential indemnity)
  - CDOs → "Collateralized" Debt Obligation
  - CLOs → " Loan "
  - TRSs → "Total Return" Swaps

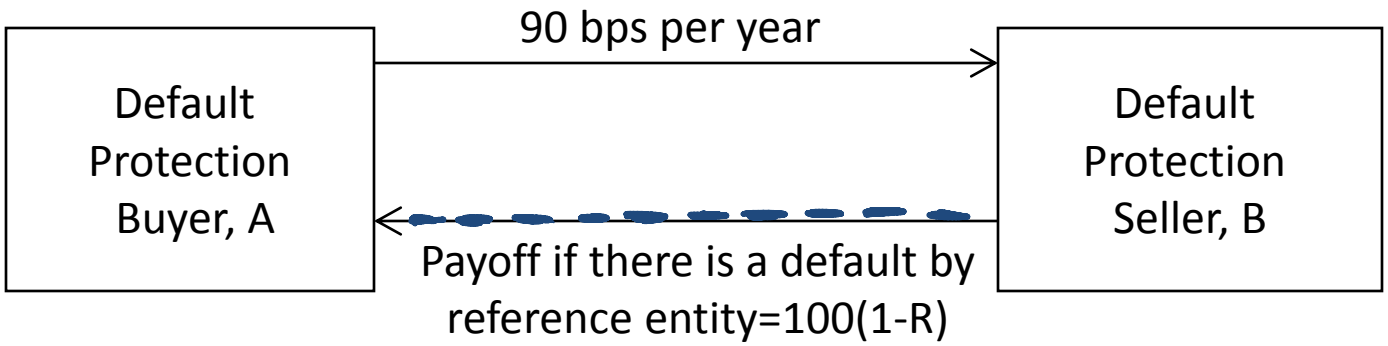




Credit Default Swaps (CDS)

- Idea:
  - Buyer of the instrument acquires protection from the seller against a default by a particular company or country (the reference entity)
  - Example: Buyer pays a premium of 90 bps per year for $100 million of 5-year protection against company X
  - Premium is known as the credit default spread. It is paid for life of contract or until default
  - If there is a default, the buyer has the right to sell bonds with a face value of $100 million issued by company X for $100 million (Several bonds may be deliverable)
- Advantages
  - Allows credit risks to be traded in the same way as market risks
  - Can be used to transfer credit risks to a third party
  - Can be used to diversify credit risks

CDS Structure

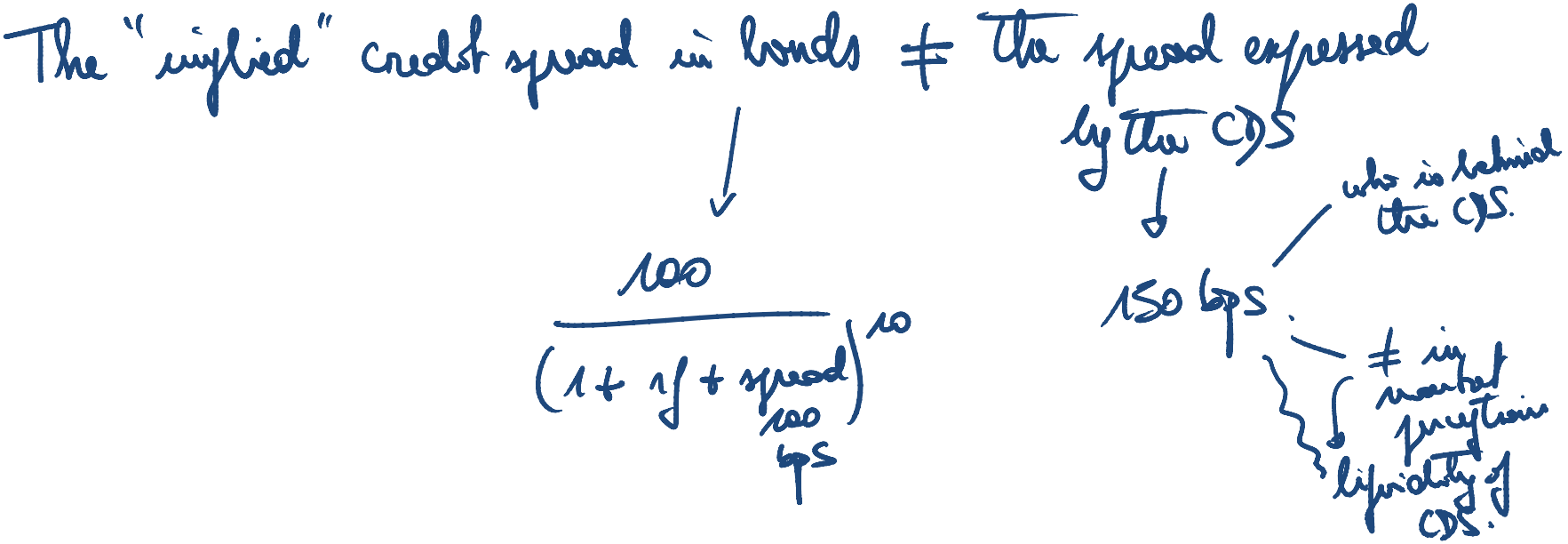


Recovery rate, *R*, is the ratio of the value of the bond issued by reference entity immediately after default to the face value of the bond

- Payments are usually made quarterly or semiannually in arrears
- In the event of default there is a final accrual payment by the buyer
- Settlement can be specified as delivery of the bonds or in cash
- Suppose payments are made quarterly in the example just considered. What are the cash flows if there is a default after 3 years and 1 month and recovery rate is 40%?

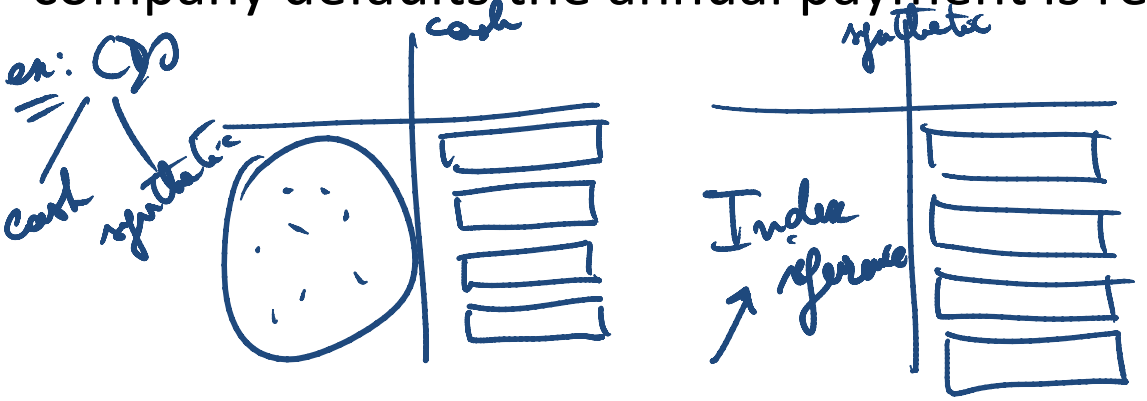
Credit Default Swaps and Bond Yields

- Portfolio consisting of a 5-year par yield corporate bond that provides a yield of 6% and a long position in a 5-year CDS costing 100 basis points per year is (approximately) a long position in a riskless instrument paying 5% per year
- What are the arbitrage opportunities in this situation if risk-free rate is 4.5%? What if it is 5.5%?



Credit Indices

- CDX IG: equally weighted portfolio of 125 investment grade North American companies
- iTraxx: equally weighted portfolio of 125 investment grade European companies
- If the five-year CDS index is bid 65 offer 66 it means that a portfolio of 125 CDSs on the CDX companies can be bought for 66bps per company, e.g., $800,000 of 5-year protection on each name could be purchased for $660,000 per year. When a company defaults the annual payment is reduced by 1/125.



CDS Valuation

- Suppose that conditional on no earlier default a reference entity has a (risk-neutral) probability of default of 2% in each of the next 5 years
- Assume payments are made annually in arrears, that defaults always happen half way through a year, and that the expected recovery rate is 40%
- Suppose that the breakeven CDS rate is s per dollar of notional principal

Unconditional Default and Survival Probabilities (Table 13.1, page 304)

Time (years)	Default Probability	Survival Probability
1	0.0200	0.9800
2	0.0196	0.9604
3	0.0192	0.9412
4	0.0188	0.9224
5	0.0184	0.9039

Calculation of PV of Payments Principal=$1 (Table 13.2, page 304)

- Payment = *s* per annum
- r_f = 5% (continuous compounding)
- Payments made at the end of each year
- Default may happen only halfway through a year

Time (yrs)	Survival Prob	Expected Paymt	Discount Factor	PV of Exp Pmt
1	0.9800	0.9800*s*	0.9512	0.9322*s*
2	0.9604	0.9604*s*	0.9048	0.8690*s*
3	0.9412	0.9412*s*	0.8607	0.8101*s*
4	0.9224	0.9224*s*	0.8187	0.7552*s*
5	0.9039	0.9039*s*	0.7788	0.7040*s*
Total				4.0704*s*

Present Value of Expected Payoff Principal = $1 (Table 13.3, page 305)

Time (yrs)	Default Proba	Rec. Rate	Expected Payoff	Discount Factor	PV of Exp. Payoff
0.5	0.0200	0.4	0.0120	0.9753	0.0117
1.5	0.0196	0.4	0.0118	0.9277	0.0109
2.5	0.0192	0.4	0.0115	0.8825	0.0102
3.5	0.0188	0.4	0.0113	0.8395	0.0095
4.5	0.0184	0.4	0.0111	0.7985	0.0088
Total					0.0511

PV of Accrual Payment made in event of a Default. Principal=$1. (Table 13.4, page 305)

Time	Default Prob	Expected Accr Pmt	Disc Factor	PV of Pmt
0.5	0.0200	0.0100*s*	0.9753	0.0097*s*
1.5	0.0196	0.0098*s*	0.9277	0.0091*s*
2.5	0.0192	0.0096*s*	0.8825	0.0085*s*
3.5	0.0188	0.0094*s*	0.8395	0.0079*s*
4.5	0.0184	0.0092*s*	0.7985	0.0074*s*
Total				0.0426*s*

Putting it all together

- PV of expected payments is
  - 4.0704s + 0.0426s = 4.1130s
- The breakeven CDS spread is given by
  - 4.1130s = 0.0511 or s = 0.0124 (124 bps)
- The value of a swap with a CDS spread of 150bps would be
  - 4.1130 × 0.0150-0.0511 or 0.0106 times the principal.

Implying Default Probabilities from CDS spreads

- Suppose that the mid market spread for a 5 year newly issued CDS is 100bps per year
- We can reverse engineer our calculations to conclude that the default probability is 1.61% per year.
- If probabilities are implied from CDS spreads and then used to value another CDS the result is not sensitive to the recovery rate providing the same recovery rate is used throughout

"Binary" CDS

- The payoff in the event of default is a fixed cash amount
- In our example the PV of the expected payoff for a binary swap is 0.0852 and the breakeven binary CDS spread is 207 bps

CDS → pays for the LGD

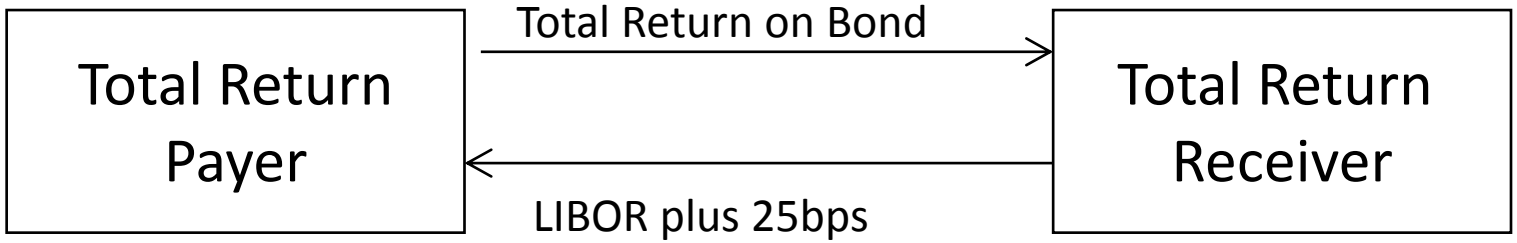
Binary CDS → 1 x amount.

CDS Forwards and Options

- CDS forward is an agreement to enter into a CDS with a specified spread at a future time
- CDS option is an option to enter into a CDS with a specified spread at a future time
- Both a CDS forward and a CDS option cease to exist if there is a default before the end of the life of the forward or option

Total Return Swap

- Agreement to exchange total return on a corporate bond (or other portfolio of securities) for LIBOR plus a spread
- At the end there is a payment reflecting the change in value of the bond
- Usually used as financing tools by companies that want an investment in the corporate bond



First to Default Basket CDS

- Similar to a regular CDS except that several reference entities are specified and there is a payoff when the first one defaults
- This depends on “default correlation”
- Second, third, and nth to default deals are defined similarly

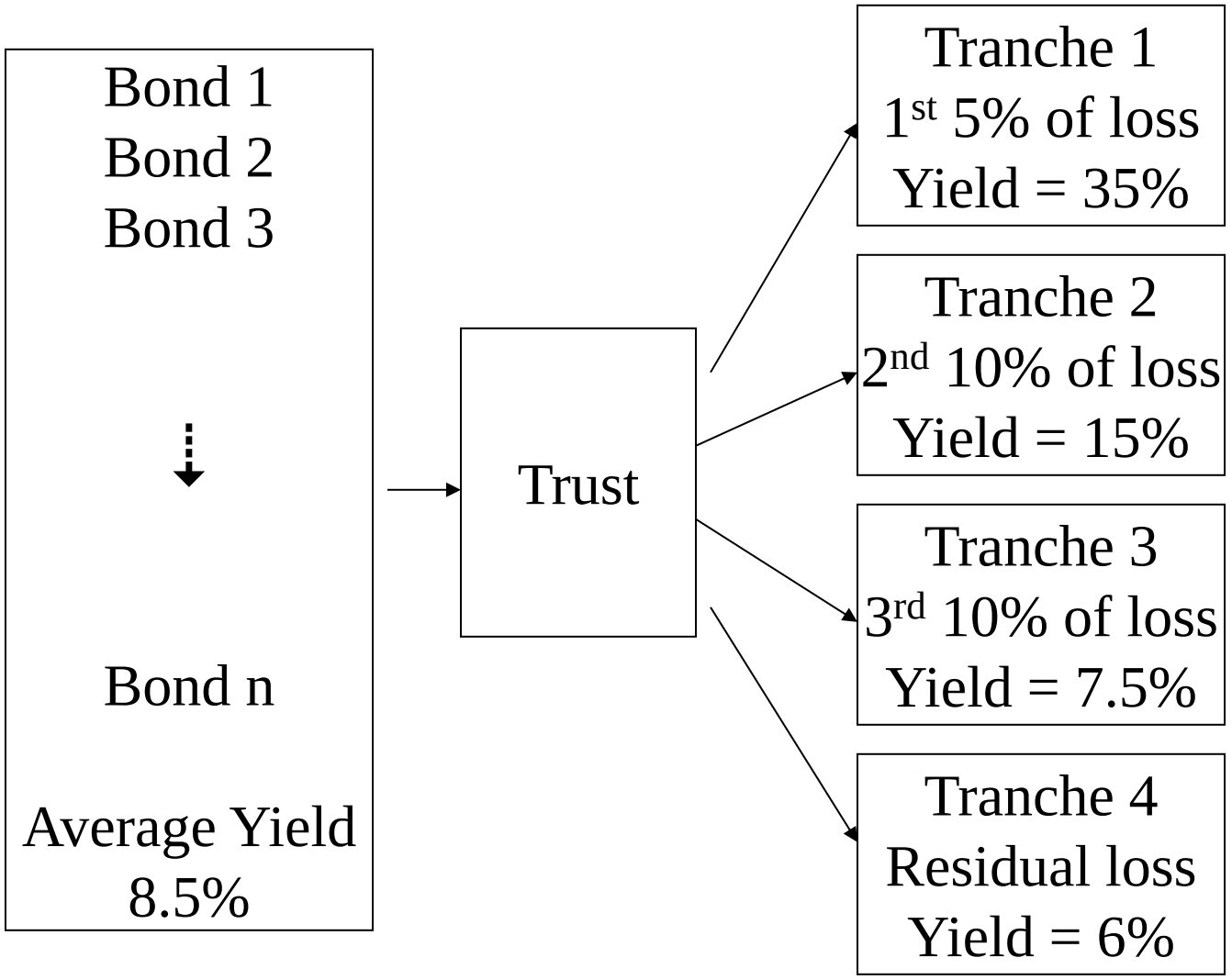
The Impact of Correlation

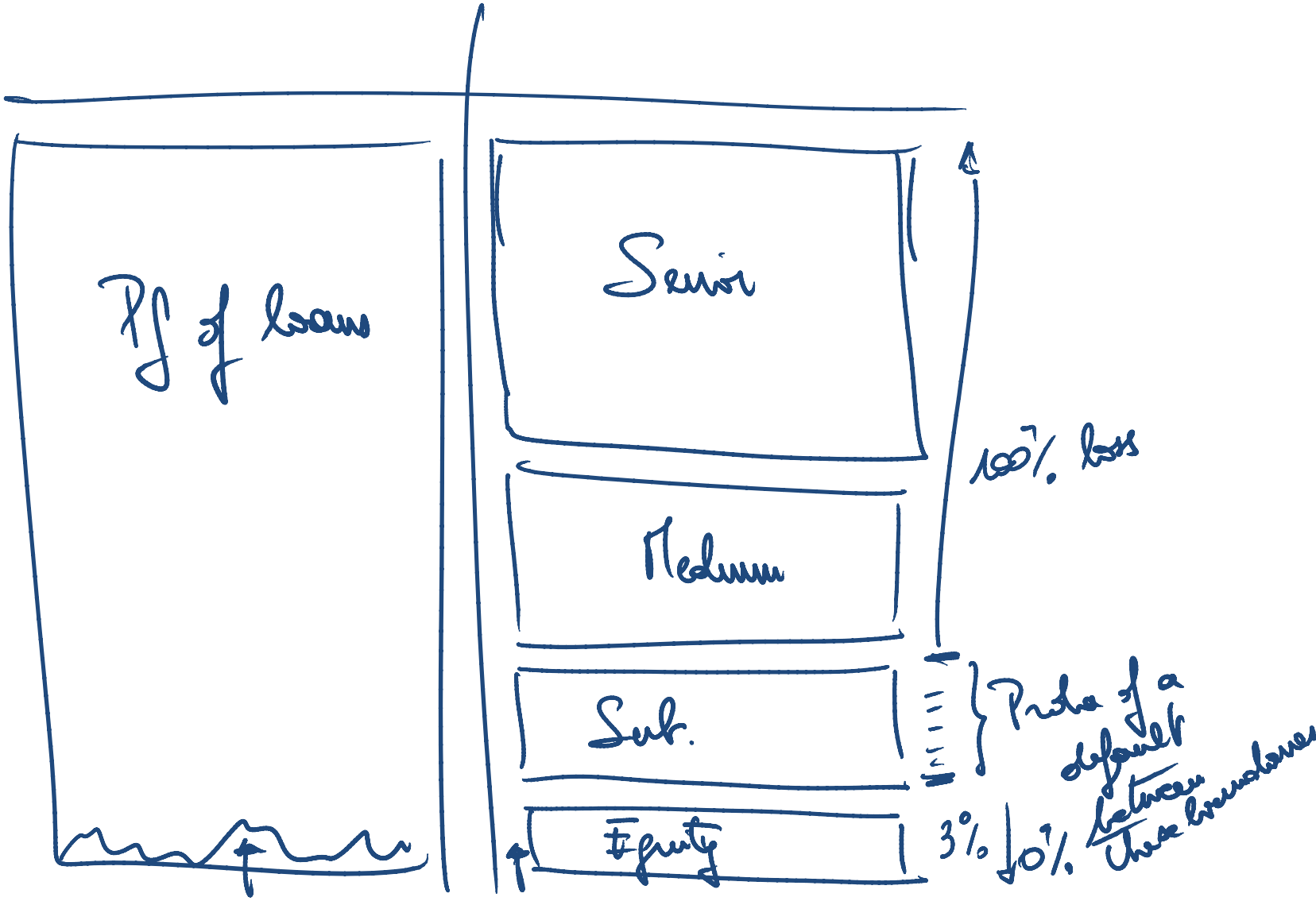
- Consider a basket of 20 names
- What happens to the cost of first-to-default protection as the default correlation increases?
- What happens to the cost of 15^{th}-to-default protection as the default correlation increases?

Collateralized Debt Obligation (CDO)

- A pool of debt issues are put into a special purpose trust
- Trust issues claims against the debt in a number of tranches
  - First tranche covers x% of notional and absorbs first x% of default losses
  - Second tranche covers y% of notional and absorbs next y% of default losses
  - etc
- A tranche earn a promised yield on remaining principal in the tranche

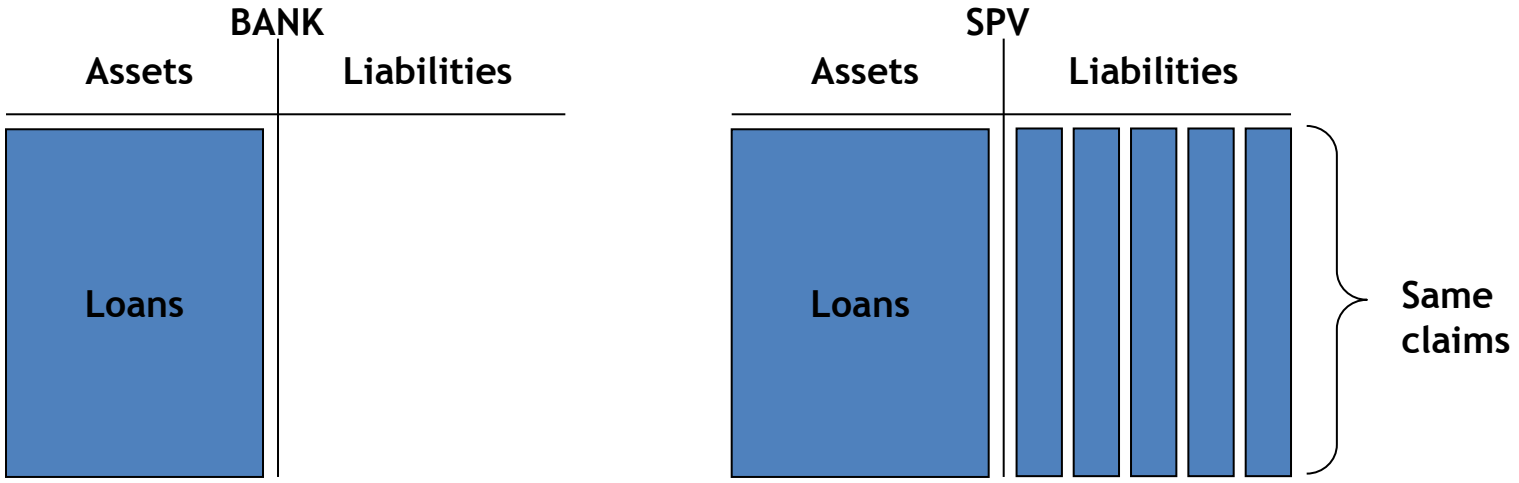
Cash CDO Structure (Figure 13.3, page 312)



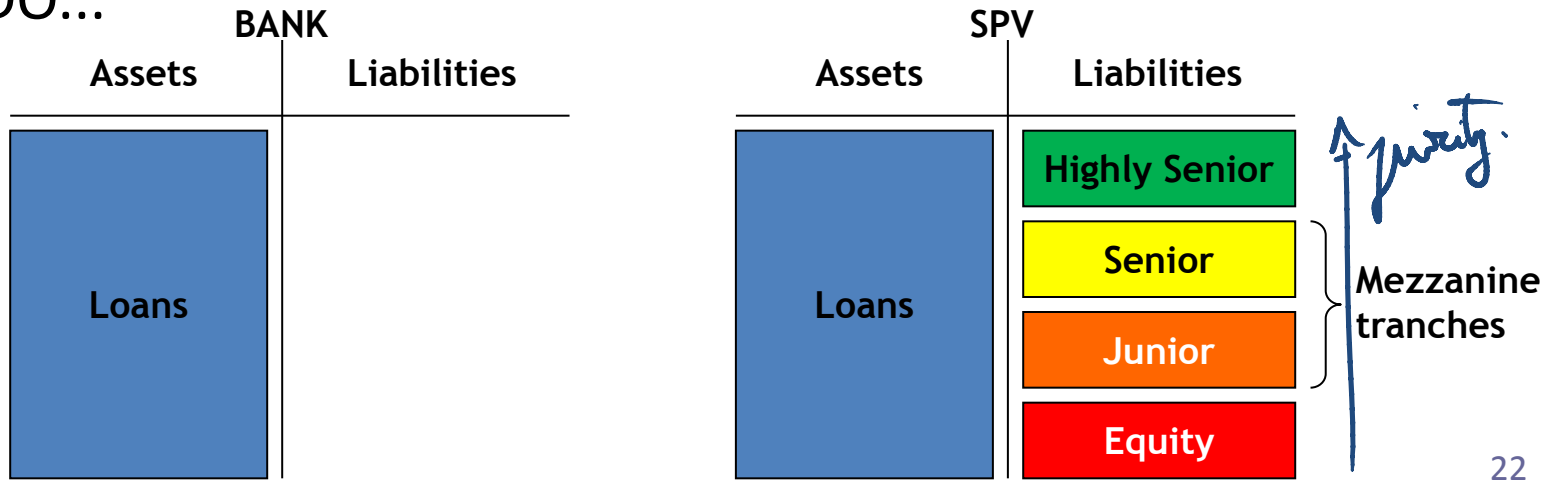

Pf of loans
Senior
Medium
Sub.
Equity
100% loss
Prob of a default between these boundaries
3%
10%

A CDO vs. other securitizations

- A standard securitisation...



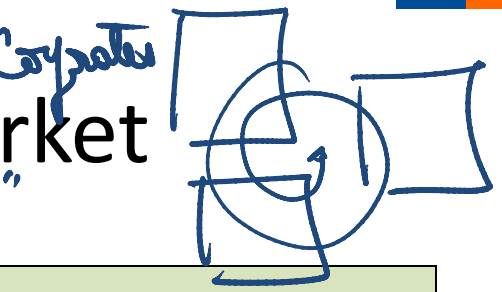
- For a CDO...



Synthetic CDO

- Instead of buying the bonds the arranger of the CDO sells credit default swaps.
- Or...any exposure through the use of Indices...

The example of CDOs' market



1. Assets

Subprime

Securitization

Q3 – Q4 2006

2. Primary Securitisation Market

AAA

AA

BBB

Watch status
April 2007

Tranches
from other
securitisations

BBB

BBB

BBB

CDO²
+ credit enhancement.

3. Secondary Securitisation Market

Re-securitization

AAA

AA

BBB

Downgrades
August 2007

4. Money Market

Bank Liquidity

Conduit

Peak Issues
9 August 2007

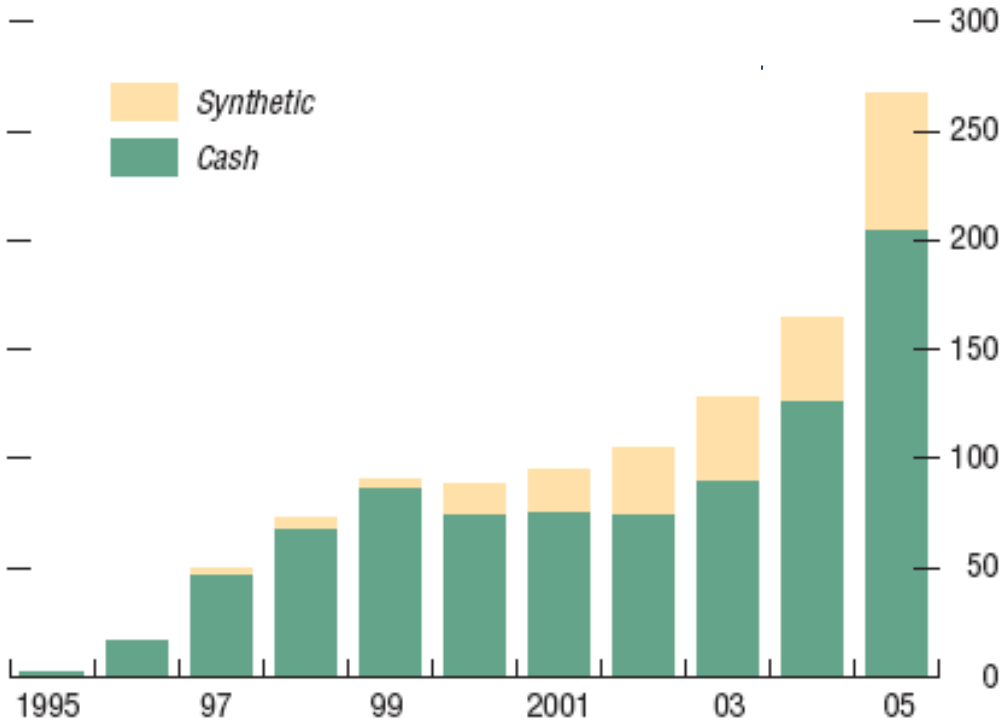
Short Term
CPs

Hedge Funds

Source : UK FSA – Securitization Conference

The market for CDOs

Figure 2.2. Global Issuance of Collateralized Debt Obligations: Cash Versus Synthetic
(In billions of U.S. dollars)



Source: Lehman Brothers.

"We offer a money-back guarantee, assuming we have the money."

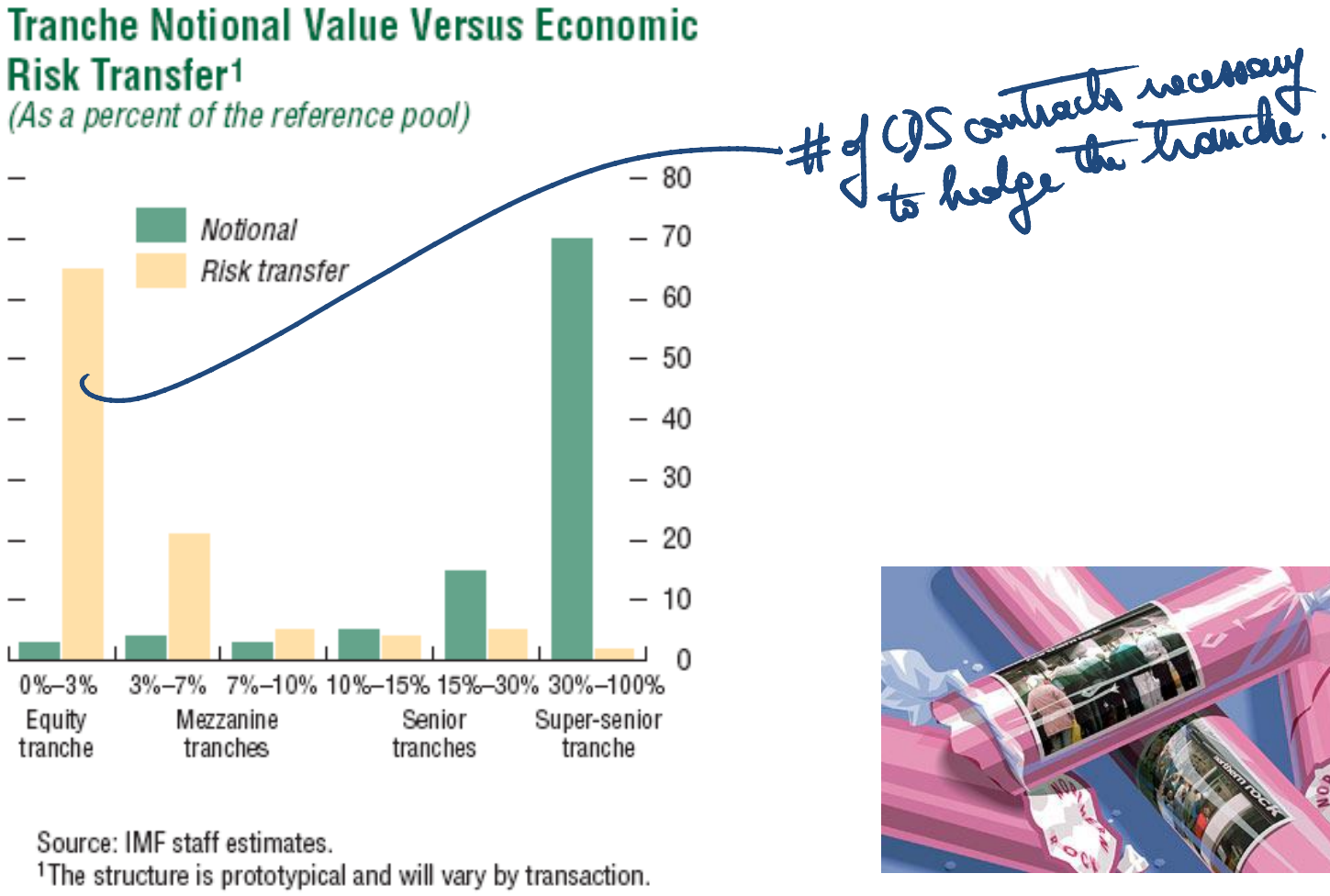
Source: Global Financial Stability Report, IMF, 2006

Responsability

"Your Honor, I would have concentrated on the facts, but they weren't in my favor!"

One particularity of financial constructions...

- Streching the risk-return relationship



Source: IMF staff estimates.
[1]The structure is prototypical and will vary by transaction.

Single Tranche Trading

- This involves trading standard tranches of standard portfolios that are not funded
- CDX IG (Aug 30, 2005):

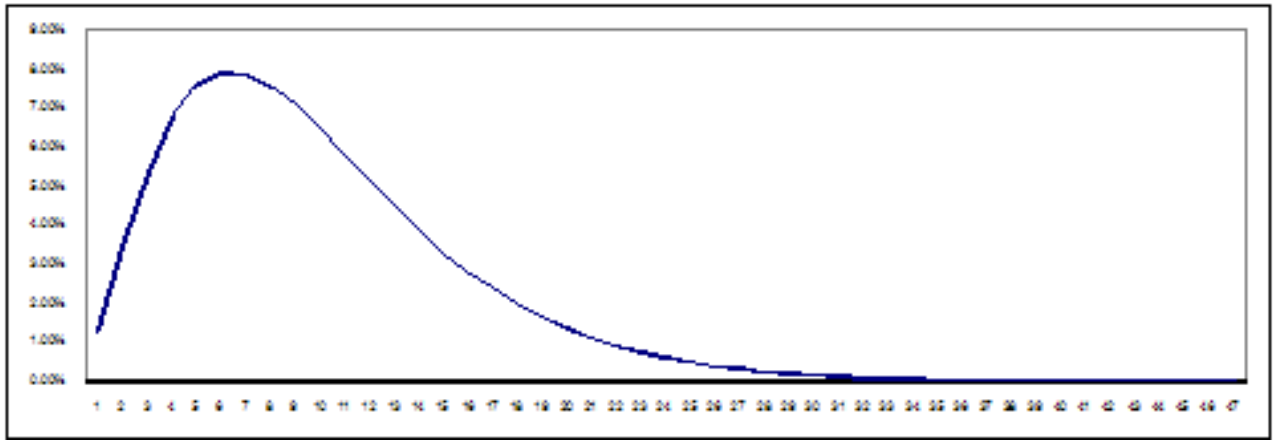
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
Quote	40%	127bps	35.5bps	20.5bps	9.5bps

- iTraxx IG (Aug 30, 2005)

Tranche	0-3%	3-6%	6-9%	9-12%	12-22%
Quote	24%	81bps	26.5bps	15bps	9bps

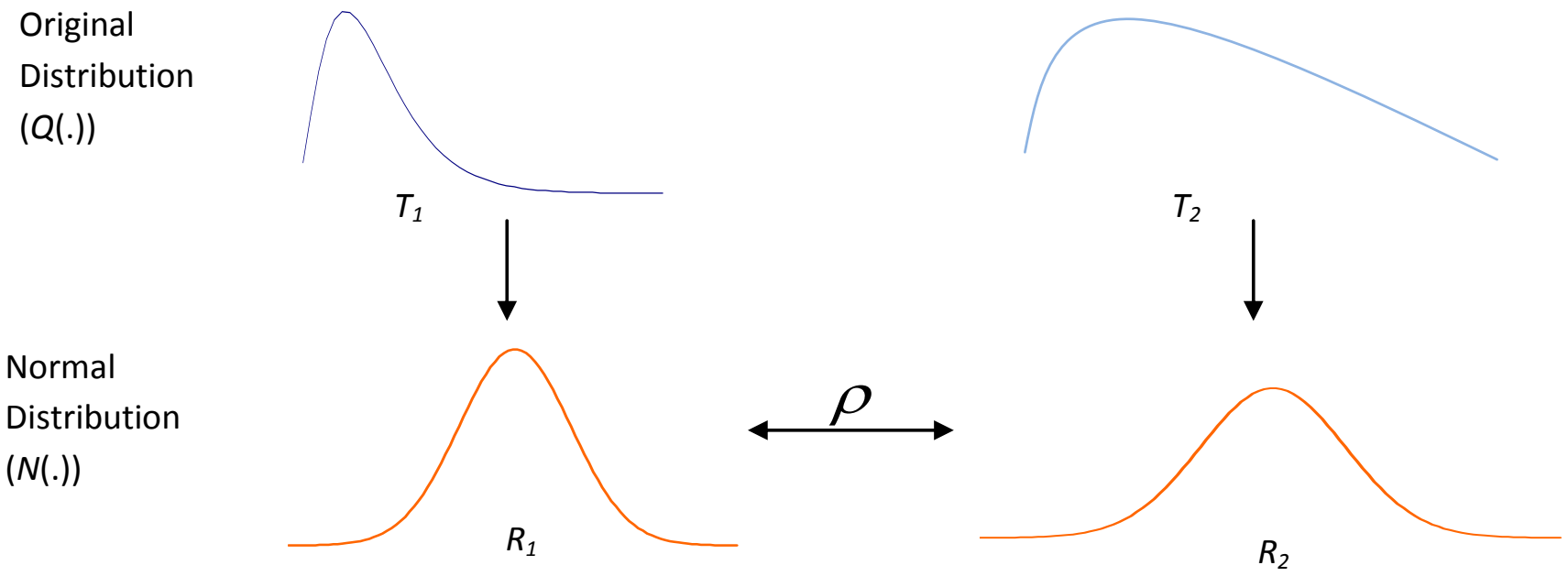
Technology (reminder)

- First we need to define the default problem as a “stopping-time” problem



Technology (reminder 2)

- How can we correlate these potential "stopping-time" functions for each counterpart?



Technology (reminder 3)

- We can use the normal distribution applied to a factor model

$$R_i = \underbrace{a_i F}_{\text{common}} + \underbrace{\sqrt{1-a_i^2}\,Z_i}_{\text{specific}}, \qquad F, Z_i \approx N(0,1)$$

- Then we can compute the probability to mature after the stopping time (uing the Gaussian copula)

$$\Pr\left(R^* < R_T \middle| F\right) = \Pr\left(T^* < T \middle| F\right) \to N\left[\frac{R_T - a_i F}{\sqrt{1-a_i^2}}\right] = N\left[\frac{N^{-1}\left(Q(T)\right) - a_i F}{\sqrt{1-a_i^2}}\right]$$

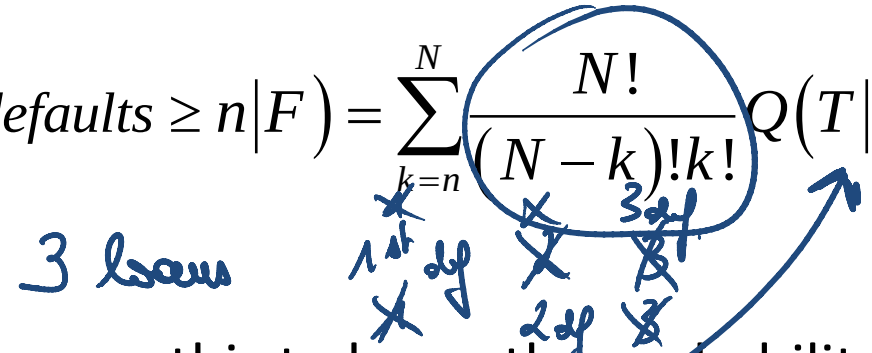
$$\Pr\left(T^* < T \middle| F\right) = Q\left(T \middle| F\right) = N\left[\frac{N^{-1}\left(Q(T)\right) + \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}\right]$$

- Hypotheses:
  - All companies assumed to have same constant default intensity that is consistent with the index, and same global correlation.
  - Define *Q(T)* as default probability by time *T*

Technology (reminder 4)

- A simple binomial model applied on this can allow us to know what is the proba of more than "n" defaults

$$\Pr\left(\#defaults \geq n \middle| F\right) = \sum_{k=n}^{N} \frac{N!}{(N-k)!k!} Q\left(T\middle|F\right)^{k} \left[1 - Q\left(T\middle|F\right)\right]^{N-k}$$



- We can use this to know the probability of a tranche to be hit and therefore to put a price on it! (conditional on *F*)

- Otherwise we use again Monte Carlo simulations!

Standard Market Model for nth to Defaults and CDOs

- Conditional on F, defaults are independent so that the probability of exactly k defaults from N companies by time T is

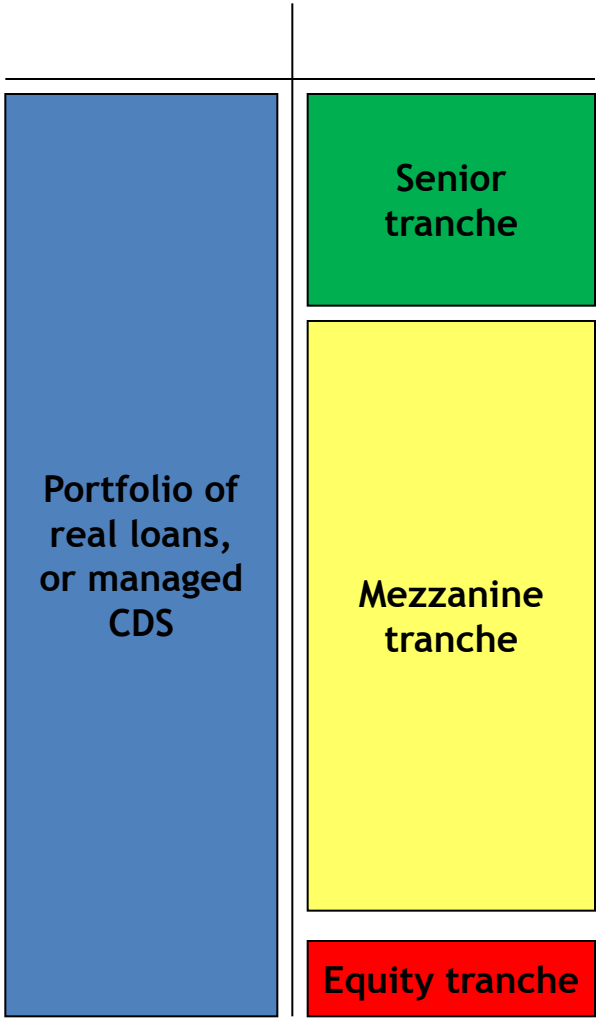
$$\frac{N!}{(N-k)!k!}Q(T|F)^k[1-Q(T|F)]^{N-k}$$

- This enables cash flows to be calculated conditional on F
- We then integrate over F
- Derivative dealers calculate an implied correlation from tranche quotes in the same way that they calculate an implied volatility from option quotes

Implied Correlations

- Tranche correlation (or compound correlation) is the correlation that prices a particular tranche consistently with market quote
- Base correlation is the correlation such that prices all tranches up to a certain level of seniority consistently with market quotes

Consequence...



What do you think is the impact of correlation in the calculation?